

DESIGN AND CALCULATION METHODS FOR HIGH-SPEED GEARS OF ADVANCED TECHNOLOGY

by

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ABSTRACT

High-speed gears of very high powers and/or very high speeds must be exactly analyzed and optimized in gearing, bearing and housing in order to achieve low noise, low vibration running with maximum safety in operation. The gearing must be checked by detailed calculations in load capacity, including an exact analysis of the scoring safety. Special design means must be applied in order to cover thermal problems at the gearing.

Besides this, the calculation and design of the plain bearings are of main interest. They also must be analyzed in detail, not only in general hydrodynamic load capacity, but also in their real temperature and pressure conditions in the oil film. The design of the plain bearing then has to be adapted to its vibration behaviour.

Using modern CAE methods, the design process can be made faster and safer. Practical examples of some of the highest-powered high-speed gears of the world prove the methods used in design, calculations and manufacturing.

INTRODUCTION

Gear units for high-speed plants are used for a wide range of applications, such as generators, compressors, pumps, gas turbines, steam turbines and motors. These drive systems must provide maximum reliability and smooth operation. Such gears are designed with single-helical or double-helical teeth. In most cases, they are running in high-precision plain bearings. High-speed gears in use today transmit as much as 70,000 kW, with circumferential speeds in the teeth of over 200 meters per second (m/s). To ensure optimum running behaviour with maximum reliability, it is imperative to analyze both the load capacity and the design of gearing, bearings, and casings. In the following, a number of items will be presented that must be considered in a modern analysis. Only a detailed examination, accompanied by high-precision manufacture and careful monitoring of the operation, will ensure the optimum running behaviour which has become a standard requirement for such plants today. Installations in the petrochemical indus-

try, for instance, must function properly in essentially continuous operation over decades.

DESIGN OF HIGH-SPEED GEARS

Design of the Gearing

In modern high-speed gears, fully case-carburized teeth are used to a great extent, providing a very high power density. A prerequisite for using this high power density is a well-adapted design of the entire gear. By utilizing fully case-carburized teeth instead of through-hardened teeth, the power to be transmitted can be nearly doubled, and/or the weight and size of the gear greatly reduced. The following relation roughly describes this effect.

The centre distance, a , is decreased in relation to the admissible Hertzian pressure, δ_{HP} :

$$a \sim 1/(\delta_{HP})^{2/3} \quad (1)$$

The admissible Hertzian pressure ratio is about 2.0; e.g., to DIN/ISO for 60 HRC case-carburized material, as compared with through-hardened teeth of 300 HB [1]. For the centre distance, the following is applicable:

$$a = 0.62a' \quad (2)$$

where

a = centre distance for case-carburized teeth

a' = centre distance for through-hardened teeth

Since the weight of the gear set, W , is

$$W \sim a^2 \quad (3)$$

the weight of the case-carburized gear set will be reduced:

$$W = 0.4W' \quad (4)$$

where

W = weight of the gear set for case-carburized teeth

W' = weight of the gear set for through-hardened teeth

On the other hand, as has already been said, the casing and the bearings must be designed to accommodate the high load capacity of the case-carburized teeth. This results in a comparative weight of the case-carburized gear approximately half that of the through-hardened gear. Such a comparison is shown in Figure 1.

The centre distance, a , and the gear weight, W , for case-carburized teeth are shown as a function of the transmitted power, P . One can recognize the interrelations illustrated: the reduced centre distance, a , and the considerably decreased weight, W , of the case-carburized teeth, as compared with the values a' and W' of the through-hardened teeth. In addition, it is evident that increased power can be transmitted by the case-carburized gears for a determined circumferential speed of the teeth. For instance, a limit circumferential speed of 125 m/s

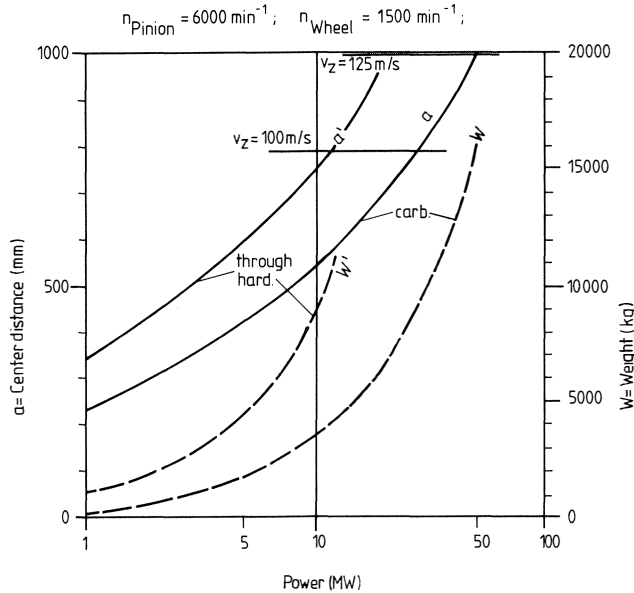


Figure 1. Centre Distances and Weights for Case-Carbureted and Through-Hardened Gears.

would mean an increase in transmittable power from 20,000 kW to about 50,000 kW. Conversely, the circumferential speed can be diminished by about 40% when using case-carburized teeth.

To make full use of the advantages of case-carburizing, the teeth must be designed for roughly the same load capacity in terms of pitting durability, bending strength, and scoring capacity. The following discussion will briefly illustrate the way in which this optimization may be achieved.

Pitting Durability

The pitting durability is calculated from the Hertzian pressure, δ_H , on the tooth flank and an admissible Hertzian pressure, δ_{Hlim} . The latter results from a strength value, δ_{Hlim} , and a safety factor, S_H . For this calculation, the influences of external loads, K_A , the internal dynamic forces, K_V , and the load distribution, K_{HB} , over the width are considered. Additional influential factors are the oil film between the tooth flanks ($Z_V \times Z \times Z_R$), and the material combination, Z_W .

Figure 2 illustrates the construction of a formula used for this calculation, according to DIN/ISO and AGMA [2]. The

<p>AGMA 218.01 Bending stress</p> $s_i = \frac{W_t \cdot K_a \cdot P_a}{K_v \cdot F} \cdot \frac{K_s \cdot K_m}{J}$ $\leq s_{at} \cdot \frac{K_L}{K_T \cdot K_R}$	<p>DIN 3990/ISO DP 6336 Bending stress</p> $\sigma_F = \frac{F_t}{b \cdot m_n} \cdot Y_{Fa} \cdot Y_{So} \cdot Y_{\epsilon} \cdot Y_{\beta} \cdot K_A \cdot K_V \cdot K_{F\beta} \cdot K_{F\alpha}$ $\leq \frac{\sigma_{Flim} \cdot Y_{ST} \cdot Y_{NT}}{S_F} \cdot Y_{SrelT} \cdot Y_{SrelT} \cdot Y_{Sx}$
<p>AGMA 218.01 Contact stress number</p> $s_c = C_p \sqrt{\frac{W_t \cdot C_a}{C_v} \cdot \frac{C_s}{d \cdot F} \cdot \frac{C_m \cdot C_l}{J}}$ $\leq s_{oc} \cdot \frac{C_L \cdot C_H}{C_T \cdot C_R}$	<p>DIN 3990/ISO DP 6336 Contact stress number</p> $\sigma_H = Z_H \cdot Z_E \cdot Z_F \cdot Z_{\beta} \sqrt{\frac{F_t}{b \cdot d_1} \cdot \frac{u+1}{u}} \sqrt{K_A \cdot K_V \cdot K_{H\beta} \cdot K_{H\alpha}}$ $\leq \frac{\sigma_{Hlim} \cdot Z_N}{S_H} \cdot Z_L \cdot Z_R \cdot Z_V \cdot Z_W \cdot Z_X$

Figure 2. Load Capacity Calculation Formulas.

durability value, σ_{Hlim} , for case-carburized teeth is 1.6 times that for through-hardened teeth when calculated by the AGMA formula, and 2.0 times that by the DIN/ISO formula.

Bending Strength

An increase in bending strength also can be observed for case-carburized teeth. Figure 2 shows the formula for bending strength calculations. The increase in bending strength is, however, not of the same order as that of pitting durability. From this, it can be concluded that the tooth size (module) should be increased correspondingly. If one uses roughly the same safety factors for bending strength and pitting durability, he will obtain an optimal module of about

$$m_n = a \cdot \frac{u}{(u+1)^2} \times (0.096 \dots 0.90) \quad (5)$$

with module m_n , ratio u , and centre distance, a , in mm.

Figure 3 illustrates this relation as a function of the centre distance. The modules calculated using Equation 5 are represented by the solid curves. The broken curves indicate the performance in actual practice. One can recognize that the module values used in practice are clearly below the theoretically calculated values. This is due to the limitation of the load capacity by scoring.

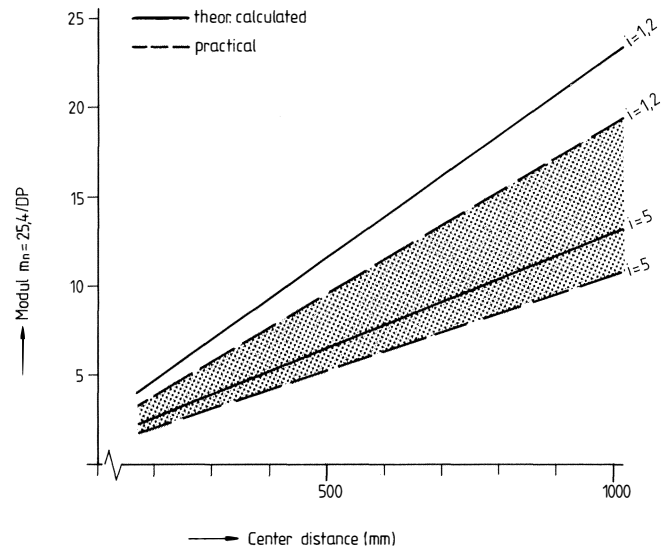


Figure 3. Relation of Tooth Size (Module) to Centre Distance.

Scoring Capacity

Scoring capacity is the factor that must be most accurately investigated in the case of such high-speed gear units. There are several calculation methods, such as the flash temperature procedure of Blok [3]. When this method is used, the instantaneous local temperature between the tooth flanks is calculated. The maximum surface temperature will be the result of the blank temperature prior to the gear mesh and the actual flash temperature. The flash temperature is calculated from the local and instantaneous amounts of load and the radii of curvature, and from the circumferential speeds and the coefficients of friction. The problem with the flash temperature method is that it does not account for all other influences, such as tip relief.

There is only one limit value of the critical temperature depending on oil viscosity; therefore, it is not possible to duly consider products such as EP oils. The new AGMA standard 421.XX [4] mentions a criterion derived from the temperature

by Blok's method, and further simplified. For this scoring index limit, values are presented for different oil temperatures and/or blank temperatures.

$$\text{Scoring criterion number} = \left(\frac{Wt}{Fe} \right)^{3/4} \times \frac{n_p^{1/2}}{P_d^{1/4}} \quad (6)$$

where

Wt = tangential load, lbs.

Fe = face width, in.

n_p = pinion speed

P_d = diametral pitch = 25.4/m

In Figure 4, this scoring index, SI , is represented as a function of the transmittable load. It can be seen that this scoring index has no fixed value for gear units in actual use, but that it increases considerably with the transmittable power. The admissible constant number of AGMA 421.XX is definitely below the values obtained from actual use. In the latest calculation methods according to Winter and Michaelis [5], the scoring capacity is based on a weighted mean integral temperature. For this calculation, a mean surface temperature is used that is calculated from the mean values of the friction coefficients and the mean dynamic loads. The decisive mean flank temperature, the so-called integral temperature, is represented from the blank temperature and a weighted integrated flash temperature.

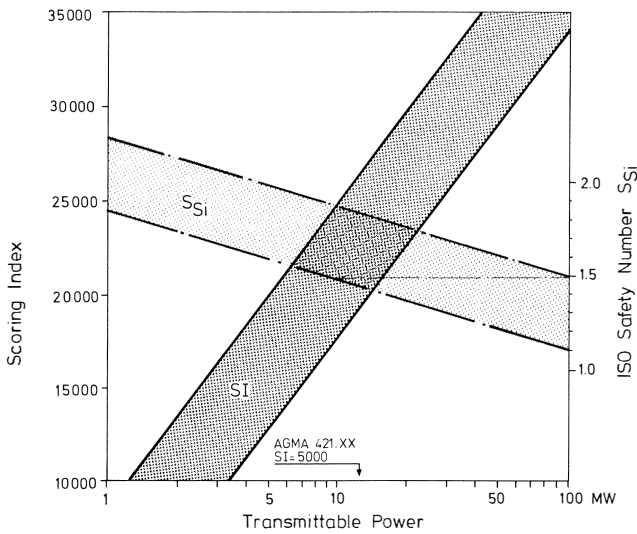


Figure 4. Scoring Index and Scoring Safety Factor Dependent on Transmitted Power for Actual Installations.

This procedure now is being adopted also by DIN/ISO. The limit values of the integral temperature are ascertained with the same procedure as the known scoring tests carried out on gear wheels. The safety, S_s , then, is the ratio of the admissible integral temperature and the actually occurring integral temperature.

$$S_{Si} = \frac{T_{ilim}}{T_i} \quad (7)$$

with

$$T_i = T_M + B \int \frac{g}{T_{BLd}} \times g = T_M + BT_{Blm} \quad (8)$$

and

$$T_i = T_{oil} + 0.7T_{Blm} + 1.5T_{Blm} = T_M + 1.5T_{Blm} \quad (9)$$

where

T_i = a result of the blank temperature before the gear mesh and a mean flash temperature.

In Figure 4, this safety value, S_{Si} , is also represented as a function of the power.

It is evident that this safety value varies only within a relatively restricted area between 2.0 and about 1.3. It can also be seen, however, that the safety factors have a tendency to be lowered as the transmittable power increases. The reason is that for large output values, there is an ever-increasing tendency towards reduced centre distance and, consequently, reduced circumferential speeds. As a result, a relatively large module must be chosen, and the risk of scoring rises. So, it is all the more important to correctly determine all influential factors: surface finish, tooth correction, material, friction coefficient, driving pinion or gear, additives to the oils, etc.

Figure 5 shows limit powers as a function of the centre distance for both bending strength, S_F , and scoring resistance, S_{Si} , for case-carburized teeth. It can be seen that, for centre distances of about 400 mm and higher, the scoring load limit restricts the transmittable power. In the case of large gears, therefore, it may be necessary to take special measures, such as surface treatment of the tooth flanks.

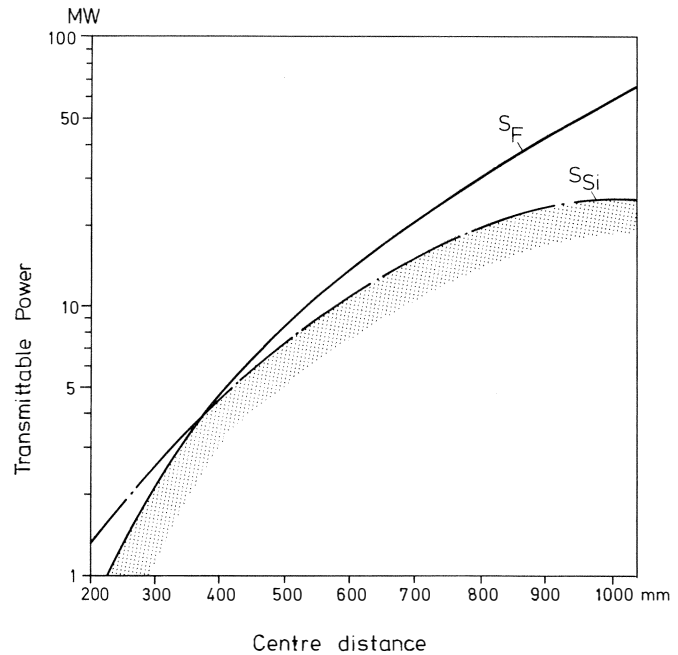


Figure 5. Power Limits in Bending Strength and Scoring Resistance.

Figure 6 shows a high-speed gear for 70,000 kW. In order to increase the tooth root load capacity and improve the running-in effect, the flanks of the wheel are copper-plated. To improve the scoring capacity by means of copper-plating, DIN/ISO provide certain reference values. In addition to this special treatment by copper-plating, tooth corrections were carried out with special care in this gear unit. The tooth corrections included relieving those areas of the tooth that are subject to scoring, and influencing the temperature variation on the tooth flank.

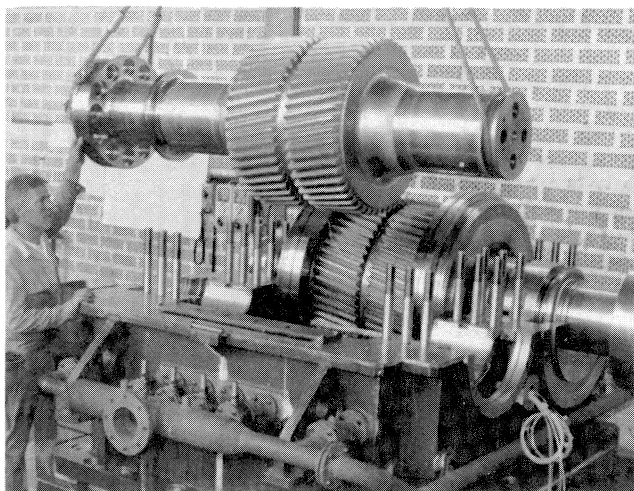


Figure 6. High-Speed Gear for 70,000 kW at Assembling for Use at a Gas Turbine Power Station.

Figure 7 illustrates the calculated temperature variation over the entire zone of contact, with and without tooth corrections. A reduction of the peak temperatures is clearly visible for properly performed tooth corrections.

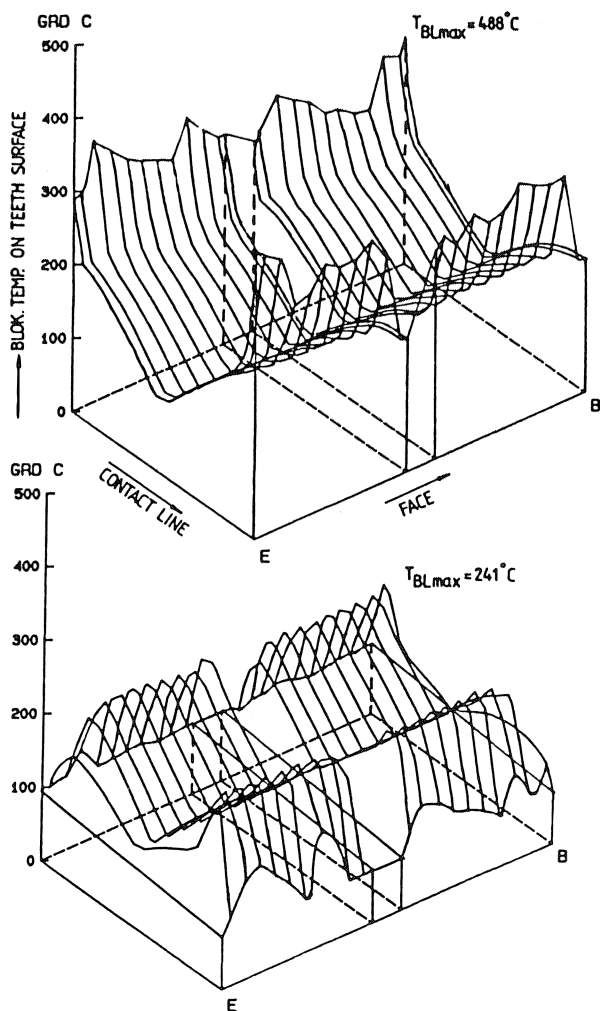


Figure 7. Calculated Distribution of Flash Temperature Without and With (Lower Part) Tooth Correction for the 70,000 kW Gear.

Bearing Design

Design of the high-speed plain bearings is an essential factor in gear unit optimization. Bearings must be designed for maximum load capacity and for optimum vibration.

The high-speed pinion shaft must be investigated to assure the elimination of unbalance-excited vibrations and self-excited oil-whip vibrations. The damping effect of the plain bearings determines the vibration behaviour with respect to the self-excited "oil-whip" vibrations. This damping behaviour depends on the type of bearing; i.e., the bearing bore design. The types of bearings used are cylindrical, two-lobe, four-lobe, and tilting pad. Extensive computing programs are available for the calculation of the damping behaviour, which allows calculation of the so-called stability border speed. Self-excited oil whip vibrations may occur above this border speed. For a specific bearing type, this stability border speed depends on the relative load, So_K , and the relative shaft stiffness, μ .

Figure 8 illustrates this stability border speed, with regard to the lateral natural frequency (with rigidly supported shaft), as a function of the load for a μ -value of 0.1, which, roughly, will occur in practice. One can see that the cylindrical bearing shows the most reduced stability border speed over the entire load range. The values given by the two-lobe and four-lobe bearings are already considerably improved, while the tilting pad bearing has extremely stable border speeds.

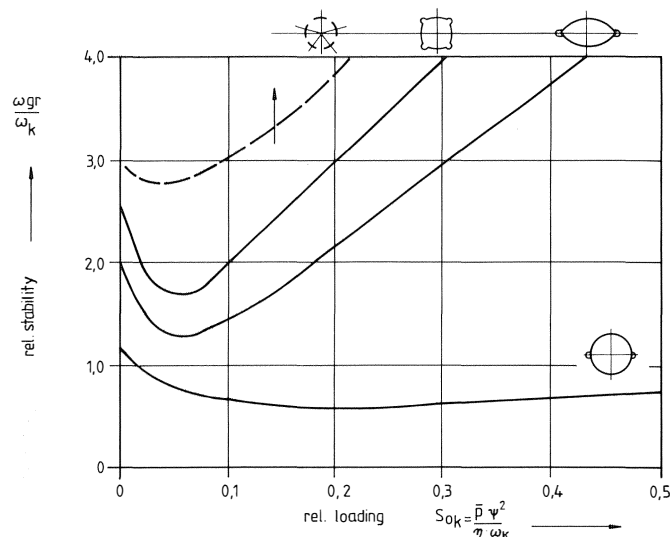


Figure 8. Stability Limits in Oil Whip Vibrations in Relation to Bearing Type.

Apart from the check of the stability border speed for oil-whip vibrations, running behaviour must be investigated for unbalance-excited vibrations; i.e., the lateral natural frequencies of the various lateral vibration modes are to be ascertained. In this regard, the stiffness of the bearing, including the oil film effect, is of decisive importance. For this problem, too, there are extensive computing programs available. The lateral natural frequency with rigidly supported shafts is reduced by the resilience of the bearings. It is imperative that there be sufficient distance between the operational speed and this reduced lateral natural frequency. Limit values are given for the admissible vibration amplitude under operational conditions; e.g., according to API 613. This problem will be further dealt with below.

When analyzing the load capacity of the bearings, the hydrodynamic quantities must be precisely determined. These are the oil film thickness, and the film's temperatures and pressures. When considering the relative oil film thickness (oil

film thickness compared with half of the bearing clearance) as a function of the referred load (Figure 9), one recognizes that the conditions here are roughly contrary to those for the damping behaviour. In other words, bearings with high damping values have relatively reduced oil film thicknesses. For example, it may appear desirable to use a four-lobe bearing because of its high stability limit. However, the reduced oil film thickness might lead to an overload of the bearing. A favourable compromise must be reached, perhaps by use of a two-lobe bearing by a modification to the shaft.

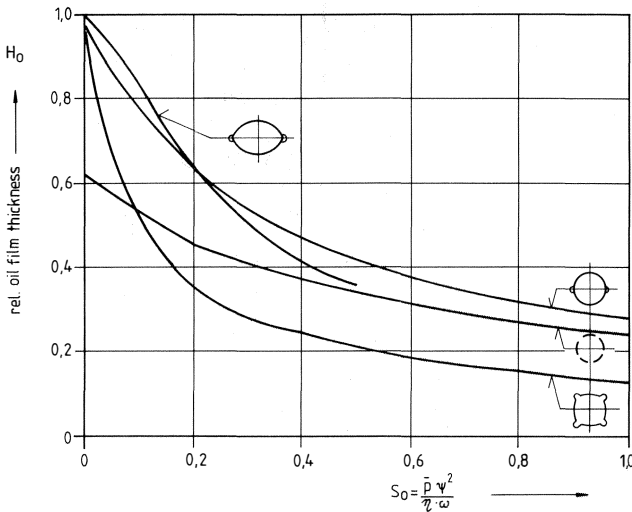


Figure 9. Relative Oil Film Thickness for Different Bearing Types.

The calculations for vibration and load capacity, as set forth above, are based on certain simplifying hypotheses, such as linearized stiffness and damping constants. In the light of the high circumferential speeds presently used, further influences on the oil film, such as turbulence and mass inertia forces, must be properly considered. According to Glienicke [6], for the calculation of the static and dynamic code numbers of such high-speed plain bearings, the modified Reynold value $R_e^* = R_e \times \gamma$ is determined, and the change in the oil film properties due to the influence of the mass inertia is calculated using R_e^* larger than 1. In the area of large R_e^* values, Taylor eddies and turbulent oil film streams occur. Because of this, the power loss and the load capacity of the oil film will mount. In an extensive computing program [7], the Navier-Stoke and/or Reynold equations are solved, and the pressure distribution in the oil film is determined, giving due consideration to the turbulent streams at very high circumferential speeds. In this calculation, the temperature variation in the oil film also is defined.

Figures 10 and 11 illustrate the temperature and pressure variations along the periphery of the bearing. The calculated values are compared with measured values obtained using thermocouples. There is relatively good agreement with the measured values. The differences can be explained by the fact that the positions of the thermocouples over the considerable width do not exactly correspond to the calculating points. Limit values from executed high-speed plain bearings can be discovered from peak temperatures and the temperature variation. Here, it is necessary to appreciate not only the peak temperatures but also the whole temperature variation (gradient) along the circumference. Bearings with peak temperatures of over 120°C have proved their usefulness in continuous operation, and extreme values of up to 148°C have been noted. These values occurred in bearings with circumferential speeds

TWO LOBE BEARING

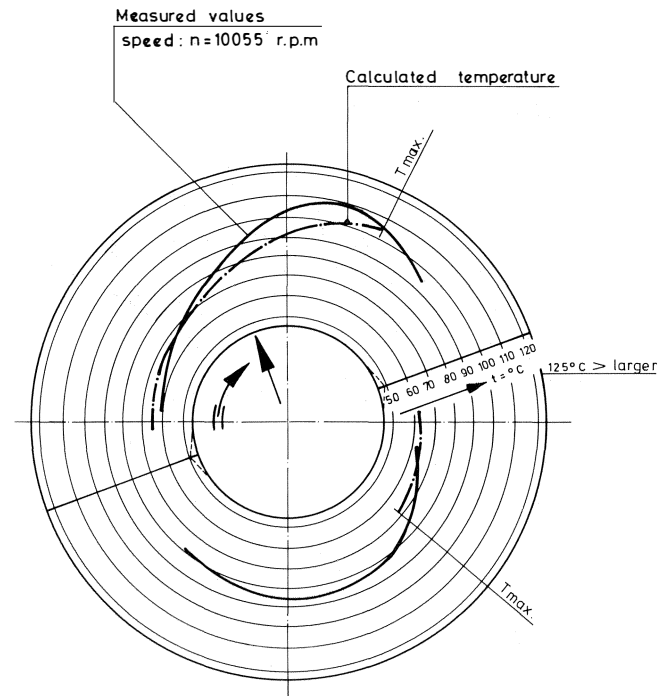


Figure 10. Temperature Distribution in a Sliding Bearing of a High-Speed Gear of 11,700 kW at 10,055 rpm.

over the range of 100 to 110 m/s.

In addition to determination of the oil film thickness, as shown above, the pressure variation (Figure 11) also is used in determining the load capacity of the plain bearing. Beyond certain critical pressure peaks, lasting deformations may occur in parts of the bearing material. Reference values for permissible bearing data have been ascertained from numerous executed high-speed gear units. These are used to safely design new high-speed gear units. The numbers shown in Figure 11 are typical for such applications.

Gear Housing Design

The high power density of case-carburized gear units must also be considered for the design of the gear unit casing, which must achieve high dynamic and static stiffnesses. Relatively extensive calculations are needed in order to determine the stresses in the casing. In difficult cases, the calculations may require modern methods such as finite element analysis.

Figure 12 shows an example of such a gear unit casing as a finite element structure. The result obtained is to be checked to ensure that the distortions under static load are sufficiently small and symmetrical. It is necessary to investigate not only the static, but also the dynamic behaviour of the gear unit casing. In doing so, the individual plates may also be checked for natural frequencies using the finite element method, in order to avoid dangerous vibrations of the steel plates. In addition to such primary vibration problems inherent in parts of the gear unit casing, the secondary vibration behaviour of the gear unit casing must be checked. Here, the vibration behaviour is especially decisive. Precise damping measures can contribute considerably to optimizing the vibration and noise behaviour of the entire gear unit. Special-design, double-walled parts filled with damping material provide a considerable damping effect.

TWO LOBE BEARING

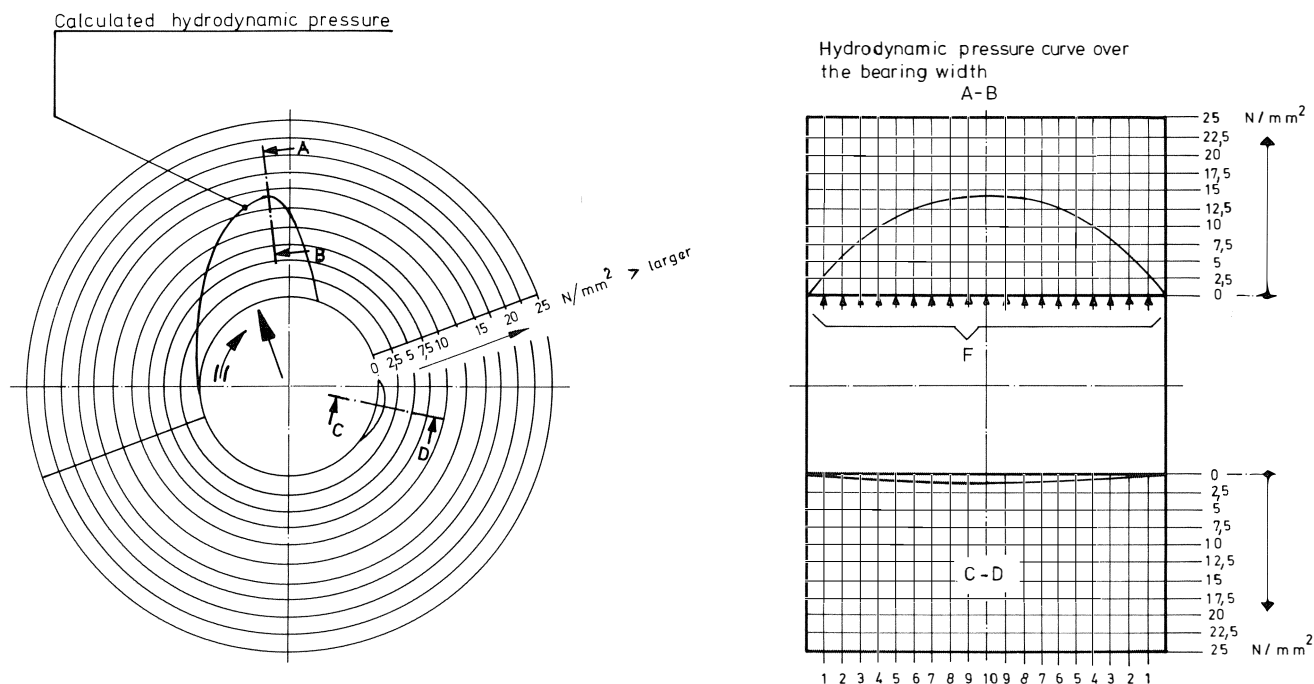


Figure 11. Hydrodynamic Pressure Distribution over Circumference and Width of the Bearing.

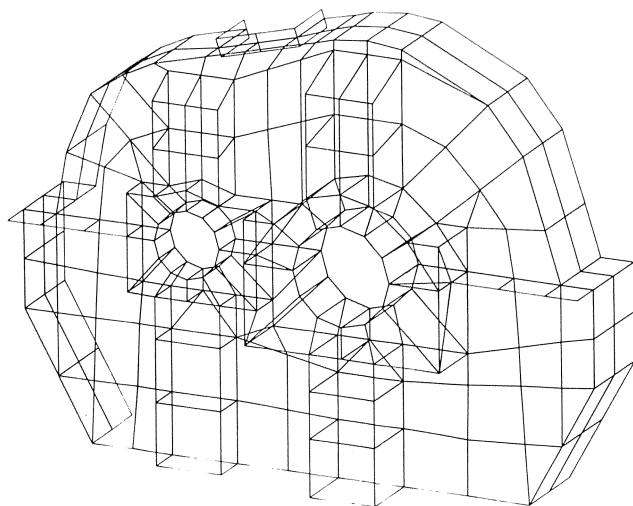


Figure 12. Finite Element Structure of Gear Housing for Stiffness Calculations.

Built gear units such as the one presented in Figure 6 (a gear unit for transmission of as much as 70 MW) testify to the suitability of the measures taken. Its noise level of 95.8 dBA sound pressure level at 55 MW is at the lower level of noise values that have yet been achieved for such power output values.

OPERATIONAL BEHAVIOUR

In this section, experience in practical service with gear units will be discussed, especially with regard to the operational reliability and the noise and vibration behaviour.

Gear Unit Noise

The gearing design, as outlined under *Design of the Gearing*, is also optimized in terms of vibration behaviour. To achieve this, special tooth corrections are necessary. Such corrections must provide relief to that part of the tooth that comes into the gear mesh first. Therefore, the design of the tooth corrections in terms of noise optimization is roughly parallel to the design of tooth corrections with regard to scoring capacity. The result must be favourable noise behaviour of the gear unit under full load conditions.

Vibration Behaviour

As stated under *Bearing Design*, the bearings are rated to achieve low lateral vibration values. High damping effects must be achieved. Also, the complete system—shaft, casing, and bearings—must be designed so that small residual unbalance does not produce important transverse vibration amplitudes. According to API, the admissible shaft vibration amplitude is

$$s = \frac{25.4}{2} \sqrt{\frac{12000}{n_1}}, \text{ in } \mu\text{m} \quad (10)$$

For a balancing quality, Q , to equal 1, this means that a specific magnifying factor, f , can be accepted:

$$f = 0.145\sqrt{n} \quad (11)$$

Figure 13 illustrates the admissible magnifying factors for balancing qualities of $Q = 1$ and $Q = 2.5$. The balancing quality, Q , is defined as speed of the centre of gravity, v_s , of the theoretical eccentricity of the centre of gravity, e :

$$Q = v_s = e \omega, \text{ in mm/s} \quad (12)$$

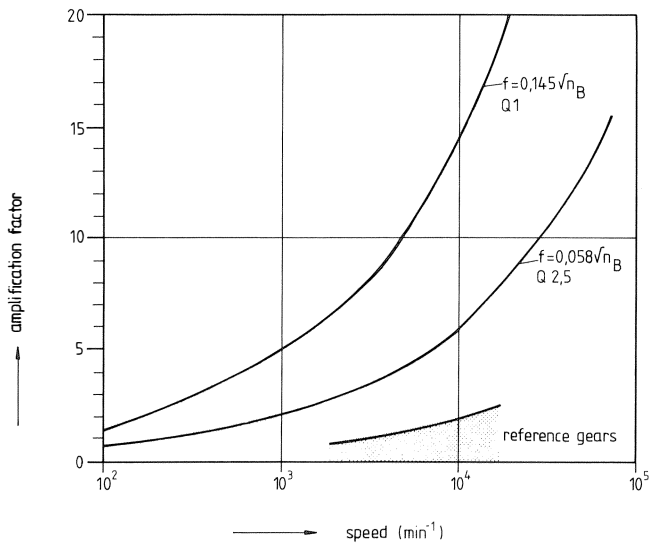


Figure 13. Amplification Factors for Lateral Vibrations Due to Unbalancing.

Achievable magnifying factors for well-designed and properly manufactured gears are indicated.

GEAR UNIT INSTALLATIONS

Figure 14 illustrates a high-speed gear for the first air-storage power plant in the world. The gear is single-helical and fully case-carburized, and its thrust collar technology has a rated output of 45,000 kW at speeds over the range of 3,000 to 7,600 rpm. A circumferential speed of 151 m/s results. The thrust collar technology enables the entire arrangement to be guided by an axial bearing positioned on the low-speed side. The thrust of the teeth from the helical gearing system is received by the thrust collar, so no internal forces act on the axial bearing.

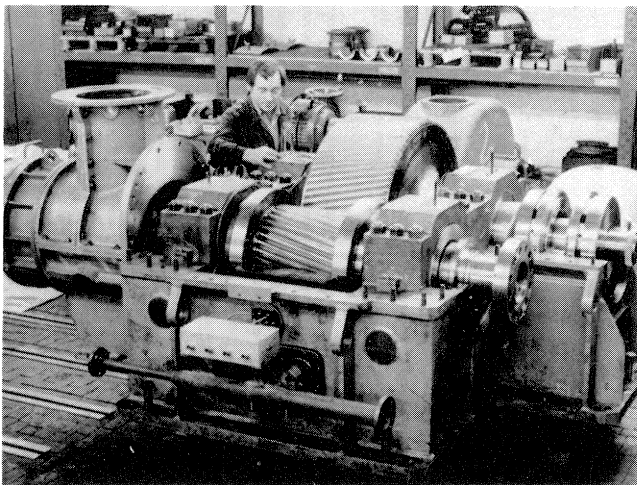


Figure 14. High-Speed Gear at Assembling for 45,000 kW at an Air Compressor Station.

Figure 15 shows another gear unit, this one for the drive of compressor plants. This unit transmits a power of 15,000 kW over a speed of 1,500 to 10,000 rpm, yielding a circumferential speed of approximately 110 m/s. It can be seen that the teeth are double-helical, and again case-carburized and ground. The casing is of welded design. A large number of these gear units

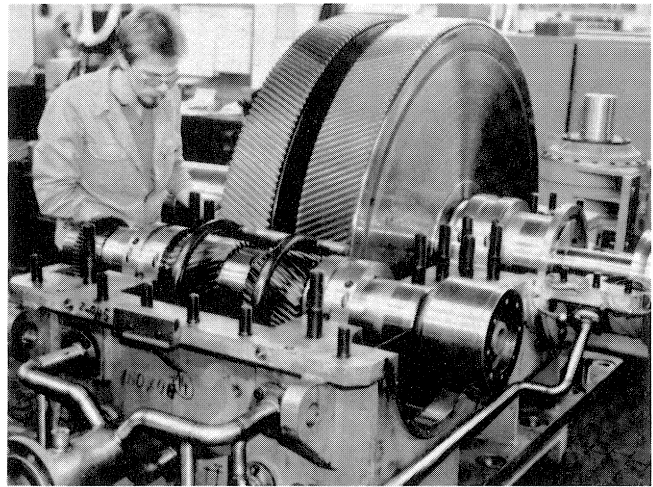


Figure 15. One of Thirteen High-Speed Gears of 15,000 kW for Gas Pipeline.

have been in operation in a gas pipeline station for many years.

CONCLUSIONS

Modern calculating methods make it possible to design extremely high-power or high-speed gears for safe and reliable operation. By precise calculations of the teeth, bearings, and casing, it is possible to achieve a favorable running behaviour. It can be supposed that the performances of such gear units must be further increased to meet the requirements of plant builders. The limits in the performances of such high-speed gear units might range today to about 120,000 kW. Such gears require thorough analysis and precise manufacture to assure that they meet service demands.

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